7057-103

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
SOFTWARE ANALYSIS AND MANAGEMENT SYSTEM

BEARING ERROR AND THE CENTRAL LIMIT THEOREM

TECHNICAL MEMORANDUM No. 32

MARC

Mathematical Analysis Research Corporation

24 August 1987

National Aeronautics and
Space Administration

JPL

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4711
ALGO_PUB_0121

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ALGO_PUB_0121	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Technical Memo 32, "Bearing Error and the Central Limit Theorem"		5. TYPE OF REPORT & PERIOD COVERED FINAL
		6. PERFORMING ORG. REPORT NUMBER D-4711
7. AUTHOR(s) Mathematical Analysis Research Corp. (MARC)		8. CONTRACT OR GRANT NUMBER(s) NAS7-918
9. PERFORMING ORGANIZATION NAME AND ADDRESS Jet Propulsion Laboratory, ATTN: 171-209 California Institute of Technology 4800 Oak Grove, Pasadena, CA 91109		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS RE 182 AMEND #187
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		12. REPORT DATE 24 Aug 87
		13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NONE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Dissemination

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Prepared by Jet Propulsion Laboratory for the US Army Intelligence Center and School's Combat Developer's Support Facility.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Bearing Error, Central Limit Theorem, Distributions, symmetry, convexity, speed of convergence

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The approach of bearing error to a Normal distribution is examined by obtaining the distribution curve for different averages of bearing error (N=1, N=2, N=3). These curves are then compared against the corresponding normal. The bearing errors are taken from a Uniform distribution, a quadratic non-symmetric and a quadratic symmetric distribution - both concave up and concave down.

7057-103

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
SOFTWARE ANALYSIS AND MANAGEMENT SYSTEM

BEARING ERROR AND THE CENTRAL LIMIT THEOREM

TECHNICAL MEMORANDUM NO. 32

MARC

Mathematical Analysis Research Corporation

EXTENDED EAAF FAMILY
ALGORITHM ANALYSIS

24 August 1987

National Aeronautics and
Space Administration

JPL

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4711
ALGO_PUB_0121

7057-103

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Bearing Error and the Central Limit Theorem

Technical Memorandum No. 32

24 August 1987

Author:

MARC
Mathematical Analysis Research Corporation

Approval:

James W. Gillis, Subgroup Leader
Algorithm Analysis Subgroup

Edward J. Records, Supervisor
USAMS Task

Concur:

for AFE

A. F. Ellman, Manager
Ground Data Systems Section

Fred Vote, Manager
Advanced Tactical Systems

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4711

PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

This specific work was performed in accordance with the FY-87 statement of work (SOW #2).

Bearing Error and the Central Limit Theorem

INTRODUCTION

Bearings are frequently computed as the average of a number of readings. Most of fixing theory assumes that the bearing error is normally distributed. Averages are usually much closer to being normally distributed than individual readings. The amount closer to normality depends on the amount of independence between readings and on the number of readings:

i) If the readings are 100% dependent, then they are the same and hence the distribution of the average is no closer to normality than the original readings.

ii) If the readings are 100% independent, then the convergence to normality is very fast as is shown in the accompanying examples. The exact speed of convergence depends on the shape of the original curve but not very much (This convergence is predicted by the Central Limit Theorem but the Central Limit Theorem does not address speed of convergence, hence the graphs provided here).

As a first approximation one may assume that some of the sources of error would be dependent and some independent. The independent error of the readings would get smaller and closer to normality as the number of readings being averaged increases. The dependent error of the readings would remain. The amount of the error that is independent could be judged using the standard deviation of the readings in comparison with the angular standard deviation. (Note: In order to compare the two, the standard deviation of the readings would need to be averaged).

THE GRAPHS

The method used to analyze the speed of convergence is demonstrated in Figure 1. The probability density functions for a Uniform distribution on the interval [a,b] are presented. Each curve represents the density function for the average of a different number of readings (n). Figure 1 has the curves for n=1, n=2 and n=3. Notice that as the number of readings being used increases, the apparent Normality of the curve is increasing.

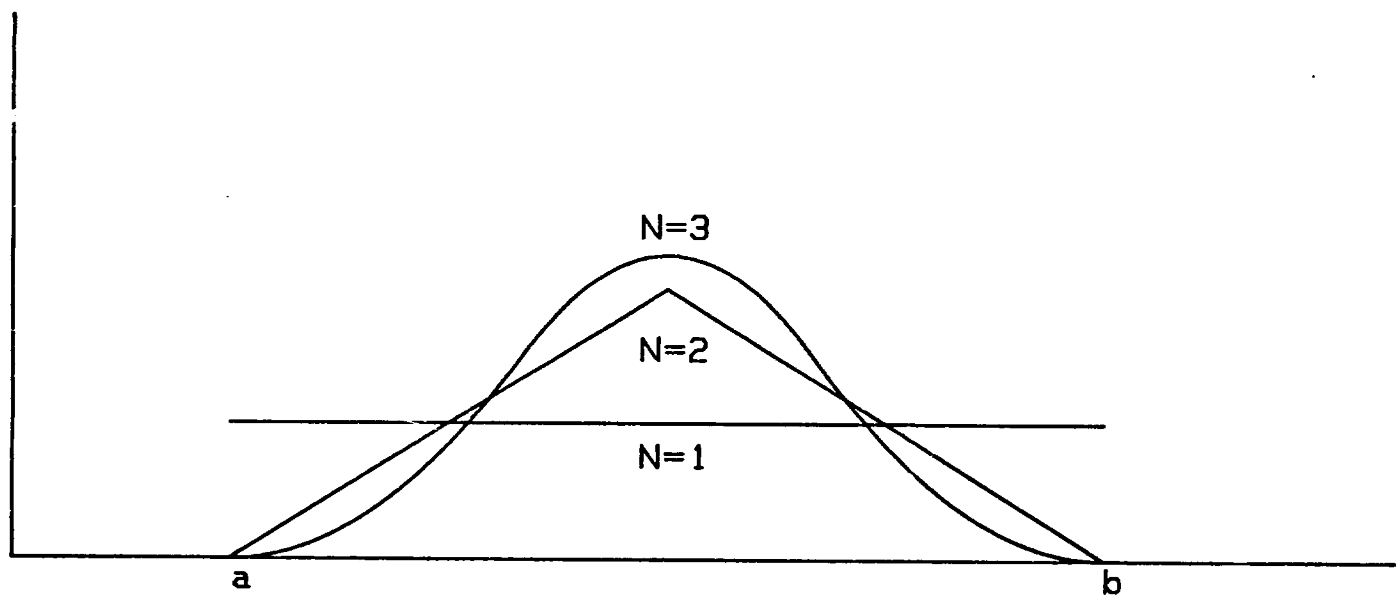
Figure 2 presents two curves. The lower curve is the density function for an average of three independent Uniform distributions. Superimposed on it is the associated Normal curve calculated using the Central Limit Theorem.

The speed of convergence for non-symmetric curves is portrayed in Figure 3. Two of the curves are the density functions of a Quadratic distribution averaging different numbers of readings. The n=1 case is almost a straight line, while the n=2 case is a decent approximation to a Normal curve. The curve superimposed on the n=2 case is the associated Normal, again determined by the Central Limit Theorem. (The Normal has mean and standard deviation determined by the quadratic equation used in the distribution.) The speed of convergence for non-symmetric density functions is not as rapid as that for symmetric ones. Comparing Figures 1 and 3 is a bit misleading since the non-symmetric example is concave down, a factor which benefits the speed of convergence.

Figure 4 presents a non-symmetric density function which is concave up. The curves pictured are for a Quadratic distribution with n=1 and n=2. Superimposed on the n=2 case is the associated Normal. Note how much slower the rate of convergence seems for non-symmetric, concave up density functions.

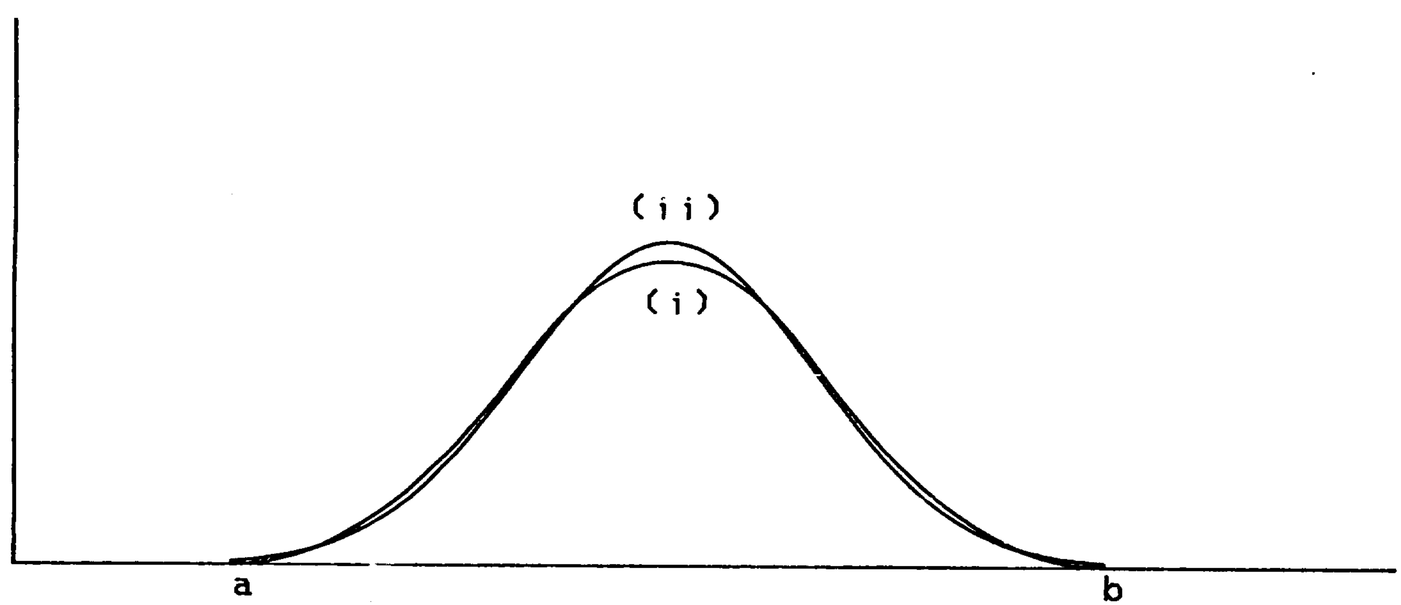
In general, symmetric density functions converge faster than do non-symmetric ones, but concavity plays a larger role. Figures 5 and 6 illustrate this problem. Figure 5 is a Quadratic distribution with a concave up density function and Figure 6 is a Quadratic distribution with a concave down density function. On both figures, the associated Normal is superimposed as a comparison with the n=2 case. The concave up density function barely resembles a Normal distribution, while the concave down density function is quite similar to its Normal.

FIGURE 1



(i) UNIFORM DISTRIBUTION; N=1, N=2, N=3 -- INDEPENDENT

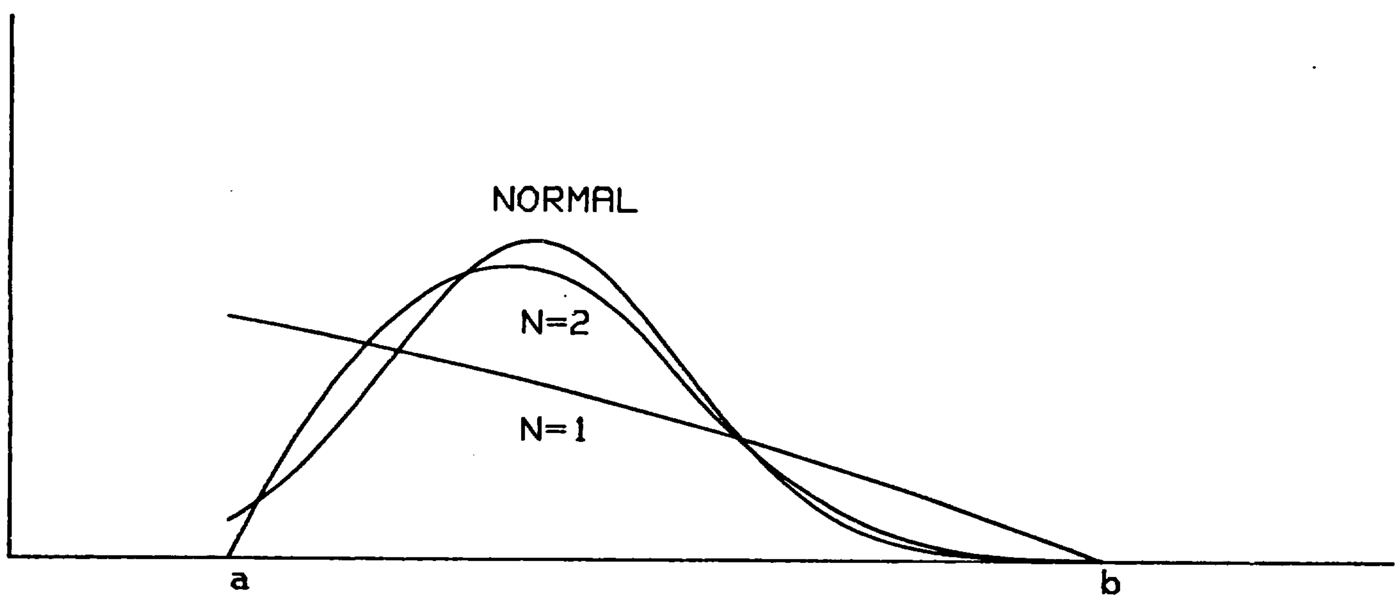
FIGURE 2



(i) UNIFORM DISTRIBUTION; N=3 -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; MEAN=(a+b)/2

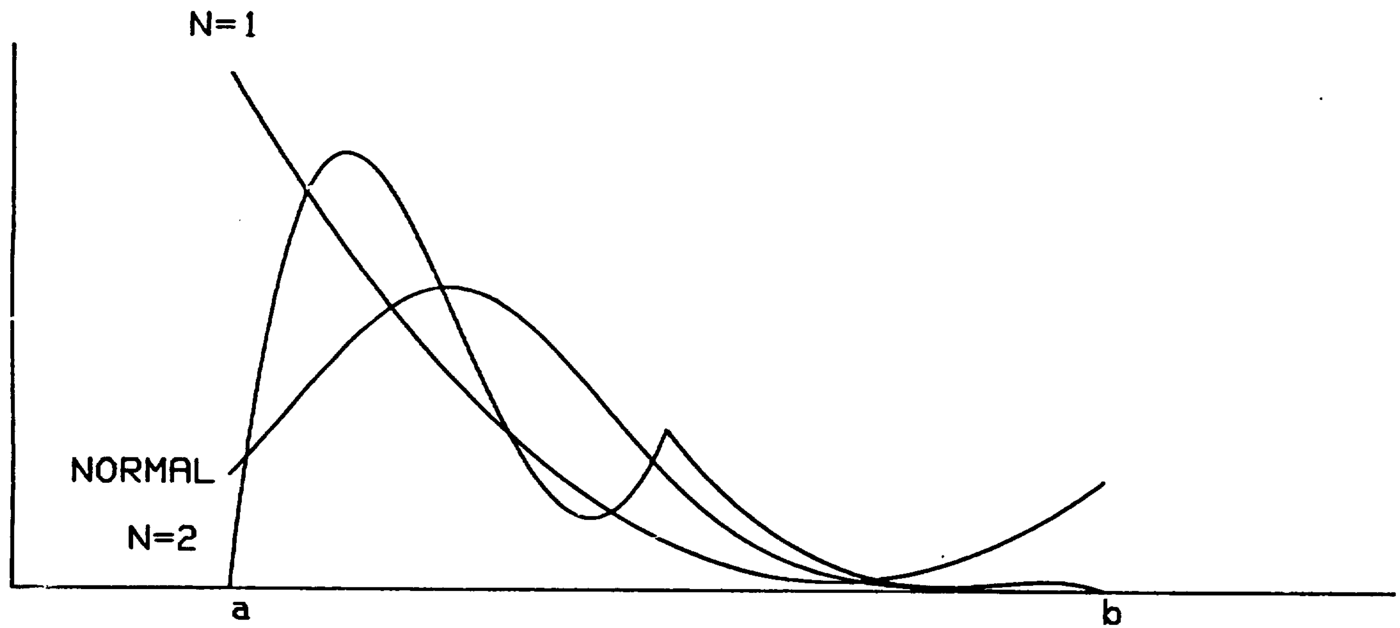
FIGURE 3 .



(i) QUADRATIC DISTRIBUTION; N=1, N=2 -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; MEAN=a+(0.35*(b-a))

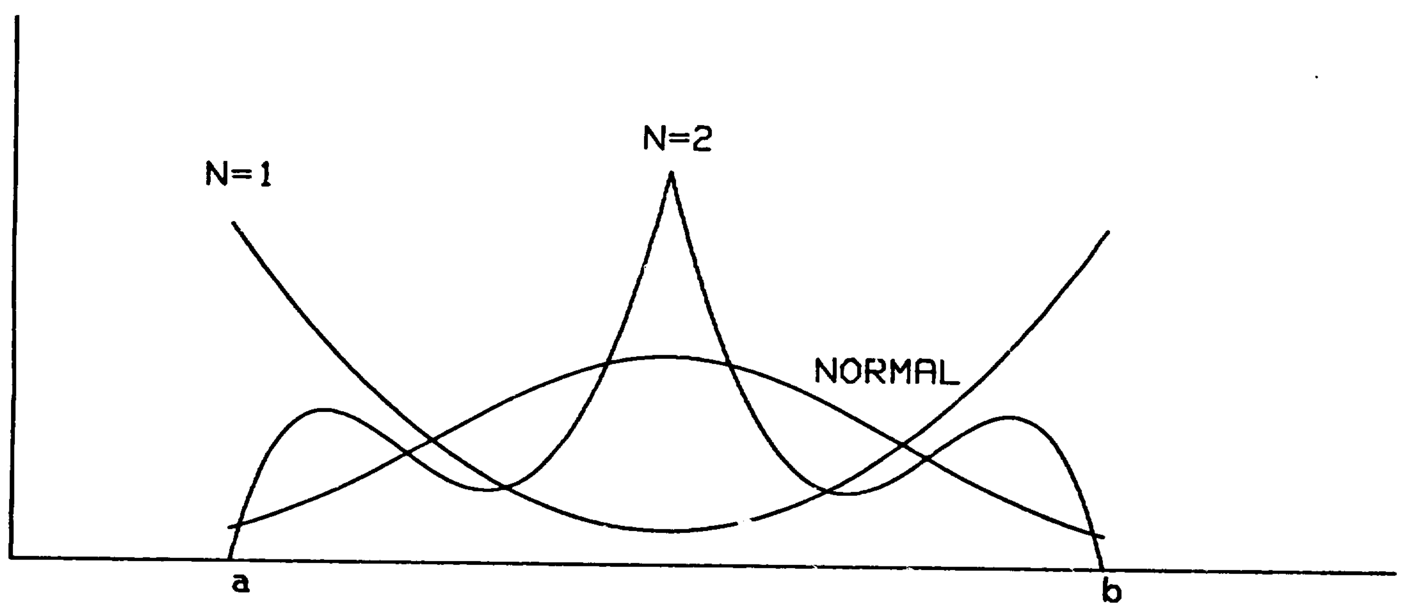
FIGURE 4



(i) QUADRATIC DISTRIBUTION; N=1, N=2 -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; MEAN=a+(0.25*(b-a))

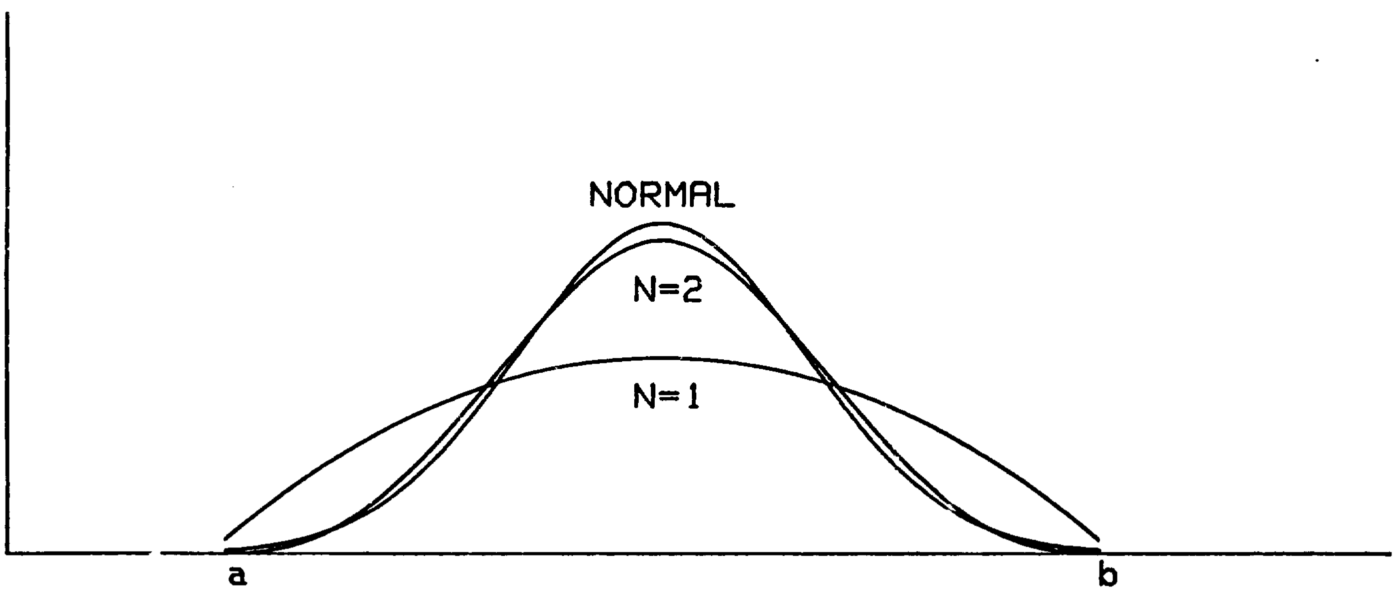
FIGURE 5



(i) QUADRATIC DISTRIBUTION; N=1, N=2 -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; MEAN=(a+b)/2

FIGURE 6



(i) QUADRATIC DISTRIBUTION; N=1, N=2 -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; MEAN=(a+b)/2